Chapter 9

Right Triangles and Trigonometry

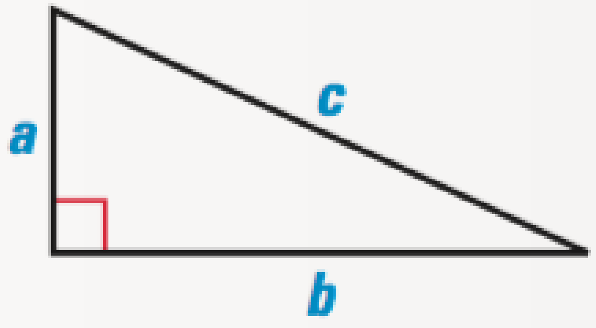
Section 2
The Pythagorean Theorem

GOAL 1: Proving the Pythagorean Theorem

THEOREM

THEOREM 9.4 ***Pythagorean Theorem***

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



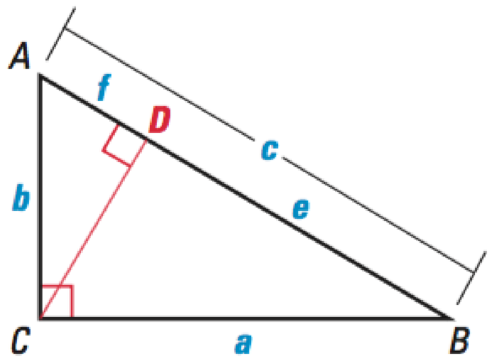
$$c^2 = a^2 + b^2$$

Proving the Pythagorean Theorem

GIVEN ▶ In $\triangle ABC$, $\angle BCA$ is a right angle.

PROVE ▶ $a^2 + b^2 = c^2$

Plan for Proof Draw altitude $\overline{CD}$ to the hypotenuse. Then apply Geometric Mean Theorem 9.3, which states that when the altitude is drawn to the hypotenuse of a right triangle, each leg of the right triangle is the geometric mean of the hypotenuse and the segment of the hypotenuse that is adjacent to that leg.



Statements	Reasons
1. Draw a perpendicular from C to $\overline{AB}$.	**1.** Perpendicular Postulate
2. $\frac{c}{a} = \frac{a}{e}$ and $\frac{c}{b} = \frac{b}{f}$	**2.** Geometric Mean Theorem 9.3
3. $ce = a^2$ and $cf = b^2$	**3.** Cross product property
4. $ce + cf = a^2 + b^2$	**4.** Addition property of equality
5. $c(e + f) = a^2 + b^2$	**5.** Distributive property
6. $e + f = c$	**6.** Segment Addition Postulate
7. $c^2 = a^2 + b^2$	**7.** Substitution property of equality

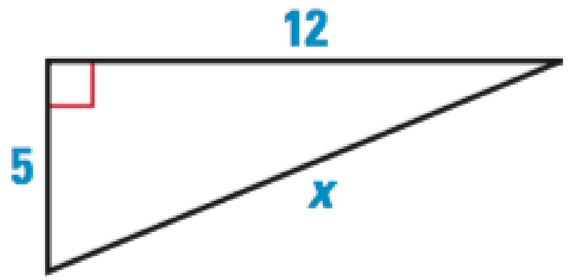
GOAL 2: Using the Pythagorean Theorem

A **Pythagorean triple** is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$. For example, the integers 3, 4, and 5 form a Pythagorean triple because $5^2 = 3^2 + 4^2$.

Example 1: Finding the Length of a Hypotenuse

$$c^2 = a^2 + b^2$$

Find the length of the hypotenuse of the right triangle. Tell whether the side lengths form a Pythagorean triple.



$$x^2 = 5^2 + 12^2$$

$$x^2 = 25 + 144$$

$$\sqrt{x^2} = \sqrt{169}$$

$$x = 13$$

yes it's a triple

LIST OF PYTHAGOREAN TRIPLES

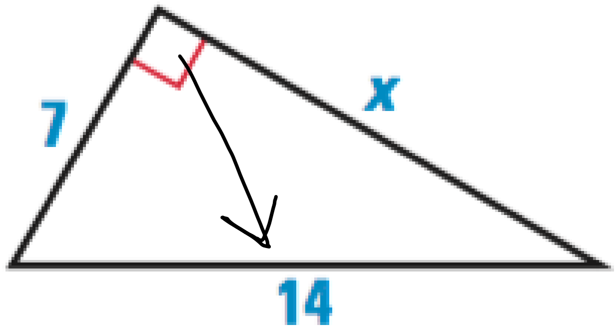
multiples of these work as well

3, 4, 5	9, 12, 15
5, 12, 13	10, 24, 26
7, 24, 25	
8, 15, 17	

Example 2: Finding the Length of a Leg

$$c^2 = a^2 + b^2$$

Find the length of the leg of the right triangle.



$$14^2 = 7^2 + x^2$$

$$196 = 49 + x^2$$

$$-49 \quad -49$$

$$\sqrt{147} = \sqrt{\cancel{x^2}}$$

$$12.1 = x$$

In Example 2, the side length was written as a radical in simplest form. In real-life problems, it is often more convenient to use a calculator to write a decimal approximation of the side length. For instance, in Example 2, $x = 7 \cdot \sqrt{3} \approx 12.1$.

$$\sqrt{147}$$

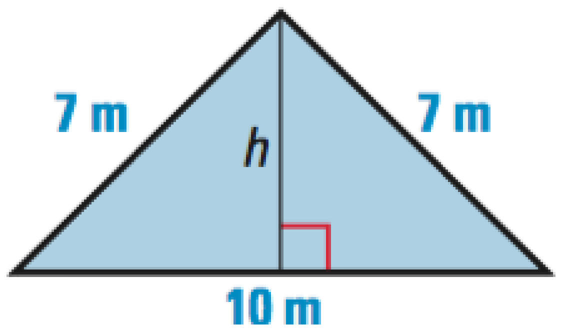
$$\sqrt{49} \quad \sqrt{3}$$

$$7\sqrt{3}$$

4   25
9   36
16   49

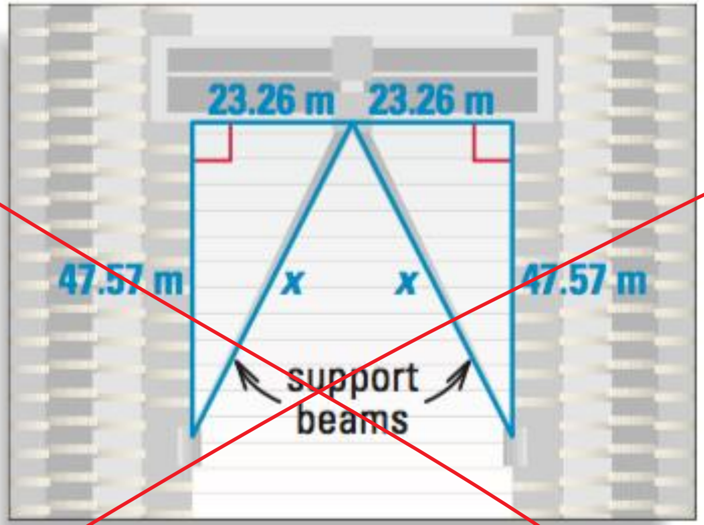
Example 3: Finding the Area of a Triangle.

Find the area of the triangle to the nearest tenth of a meter.



Example 4: Indirect Measurement

SUPPORT BEAM The skyscrapers shown on page 535 are connected by a skywalk with support beams. You can use the Pythagorean Theorem to find the approximate length of each support beam.



EXIT SLIP